A Comparative Study of Formulas for Choosing the Economically Most Advantageous Tender

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Abstract
Choosing the best bid is a significant step in any tendering process. If the award criterion is the economically most advantageous tender, this involves scoring bids on price and quality and ranking them. Scores are calculated using a bid evaluation formula that takes as inputs price and quality, and their respective weights. The choice of formula critically affects which bid wins. We study 28 such formulas and discuss several of their aspects, such as relative versus absolute scoring, ranking paradox, indifference curves, protection against an extremely high price, and how a formula reflects the weights of price and quality.

Keywords: Tendering, bid evaluation formula, bid selection

Introduction
Procurement entails the process of obtaining works, goods and services from external suppliers, needed by the procuring entity to carry out its primary and support functions (Van Weele, 2010). Professionalism in procurement is important for several reasons. First, every operation relies on a supply of inputs that are in many cases selected by procurement. Second, procurement can play a vital role in the delivery of strategic objectives. Third, efficient procurement can result in considerable monetary savings. Fourth, efficient procurement can help to achieve the best value for money. Fifth, when it involves public money, poor procurement decisions or failure to comply with procurement legislation can result in legal challenges.

Running a professional tendering process is particularly visible and relevant in the context of public procurement. Under EU procurement law, objective, transparent and non-discriminatory criteria and the relative weights for quality and price have to be published in the contract notice listed on the EU TED (Tenders Electronic Daily). In addition, the bid evaluation methodology has to be published (e.g. Mateus et al., 2010). However, the findings of this paper are not confined to the public procurement context. Private firms also need to understand what bid evaluation formulas they are using and how such formulas are different from one another.

Procurement is a process that can be divided into six phases: determining specifications; selecting the supplier(s); contracting; ordering; expediting, and finally following up and evaluating (Van Weele, 2010). This paper concerns the second phase; that of supplier selection. In this phase, qualified suppliers need to be identified, and a long list of suppliers needs to be whittled down to the supplier (or suppliers) selected for a contract (De Boer et al., 2001). Various approaches for supplier selection exist, such as face-to-face negotiations, electronic reverse auctions, or sealed bid tendering. Buyers can select suppliers based on price only, or on a combination of price and quality, the latter often being called a selection of the economically most advantageous tender (EMAT). In this paper, we are interested in approaches where suppliers bid, and the procuring entity uses EMAT to select the best bid(s). For the sake of clarity, we assume the buyer seeks to select one supplier, and therefore one best bid.

When opting for a tendering procedure based on EMAT, the buyer needs to make a number of key decisions: Which quality (i.e., non-price) criteria to include, how to score each dimension, how to weigh each quality dimension so as to come to one overall quality score, how much to weigh quality versus price, and finally, which formula to use to combine the quality score and the price into one overall score, so that bids can be ranked (De Boer et al., 2006; Mateus et al., 2010). In this paper, we focus on the latter decision: The formula that is used to combine the quality score and the price of each bid into an overall score.

Earlier studies have identified the importance of choosing an appropriate bid evaluation formula (Chen, 2008; De Boer et al., 2006; Mateus et al., 2010). According to Chen (2008), the bid evaluation formula plays a key role in (public) procurement, since it determines what ‘the economically most advantageous tender’ is. Chen (2008) focuses particularly on one aspect of (relative) bid evaluation formulas, namely the issue of ranking paradox. Using examples, Chen (2008) shows that ranking paradox is possible for relative bid evaluation formulas because the axiom of independence of irrelevant alternatives is violated (see also De Boer et al., 2006). Chen (2008) states that in practice most bid evaluation formulas used are relative.

Smith (2010) claims that buyers may end up with a sub-optimal outcome due to their misunderstanding of the bid evaluation formula’s impact on the procurement process. He points out the significance of bid evaluation formulas in the procurement process by giving examples of tenders whose outcomes would be completely different if variations of bid evaluation formulas were used. As an advice, he suggests, as does Chen (2008), that a buyer before using any bid evaluation formula should perform a simulation study of its possible outcomes. This should help to determine whether the outcomes are acceptable. Sykes (2012) stresses the need to carefully assign weights to price and quality and notes that assigning a lot of weight to quality can be costly.

In the current literature, various informative examples of unintended consequences of certain types of bid evaluation formulas are given, and a number of issues related to such formulas are discussed, such as the ranking paradox, and the price premium that may be paid if weights are not assigned appropriately. In this paper, we make the following contributions to this literature. First, we look at some additional aspects of bid evaluation formulas such as protection against an extremely high price and how a bid evaluation formula reflects weights of price and quality. Second, we use real tender data to evaluate the likelihood of a ranking paradox. Third, we critically assess the performance of 28 bid evaluation formulas collected from purchasing practice on these aspects.

Method

The formulas that are compared in this study were identified on the basis of an extensive literature review, yielding 16 formulas, and an Internet search, yielding three more formulas. A further eight formulas were provided by Negometrix, a procurement services provider, which took the formulas from real cases they came across serving clients. We simulate some of the behaviors of the formulas using real tender data. Our dataset consists of 357 real tenders from August 2001 to March 2011. The total number of bidders in these tenders is 1642. The minimum number of bidders per tender is 2 with a median of 3 and a maximum 38 bidders per tender.

Relative versus absolute formulas

There are two main approaches to the evaluation of bids: relative and absolute. In the relative approach, after all bids are submitted, each bid is evaluated using a formula that takes as one of its inputs a characteristic of the total set of bids, such as the lowest quality, the highest quality, the lowest price or the highest price. Of our set of 28 formulas, 20 are relative formulas (indicated with "R" in the second column of Appendix A). An example of a relative formula is Formula 1 in Appendix B.

An absolute formula does not utilize information from the submitted bids as a reference point. In other words, the score calculated using an absolute formula depends only on price and quality of a given bid. An example of an absolute formula is Formula 26 in the Appendix B. A practical advantage of an absolute formula is that bidders can calculate the monetary value that buyers attribute to each weighted sub criterion. This supports bidders' decisions to fulfill certain criteria or not; after all, it could cost a bidder more than the buyer's value to satisfy the criterion. This aspect is useful in guiding both buyers and bidders in preparing and submitting tenders. Another advantage of an absolute formula is that bidders can calculate their score before submitting their bid (Chen, 2008). The knowledge of the total score does not however help bidders to estimate their chances of winning the tender, as this score is only relevant in comparison with the scores of other bidders. Moreover, calculating the score is often not possible for the supplier because many quality criteria are evaluated and scored by the tendering entity only after bid submission. When a relative formula is used, bidders can only estimate their final score as it depends on the other submitted bids, which are unknown a priori.

Ranking paradox

When a relative formula is used and one or more bids with a certain quality and price are removed, the ranking of the (remaining) bids could change (Chen, 2008; De Boer et al., 2006). This issue is also studied as part of social choice theory as a consequence of selecting the best option from a number of alternatives based on individual choices of a group. While the changed ranking effect is common and known in contests (e.g. elections or sports), it seems less intuitive when ranking bids in tender procedures. This is why it has been referred to as the 'ranking paradox', a term we will also use to stay connected to other publications on the topic of procurement and bidder selection.

To obtain some idea about the impact of the ranking paradox in procurement, we analyzed the 20 relative formulas from the set of 28. We applied these formulas to the 280 tenders in our dataset with more than two bids, and generated 280 initial rankings. Then, for each tender we removed one bid and compared the initial ranking with the final ranking. After the bid ranked as number 1 was removed from the initial ranking, we compared the bid ranked as number 1 in the final ranking with the bid ranked as number 2 in the initial ranking. If these two bids are different, it means that the ranking paradox has occurred. We refer to this as 'number 1 drop-out ranking paradox'. After removing a bid ranked not as number 1 from the initial ranking, we compared the bid ranked as number 1 in the final ranking with the bid ranked as number 1 in the initial ranking. If these two bids are different, it means that the ranking paradox has occurred. We refer to this as ‘number n drop-out ranking paradox’. Likelihood of a ranking paradox is calculated as a ratio of the total number of cases that ranking paradox occurred over the total number of tenders analyzed.

Relative formulas with the ‘number n drop-out ranking paradox’ risk, create the possibility of bid rigging; A non-relevant bidder submits a bid with no intention to rank number 1 in the tender, but to influence the bid of a befriended bidder. Formulas with a ‘number 1 drop-out ranking paradox’ are not affected by the risk of bid rigging. The third and fourth columns of Appendix A show for how many of the 280 real tenders, either of these two ranking paradoxes occurred.

These outcomes are quite revealing in several ways. First, for eight relative formulas the ‘number 1 drop-out ranking paradox’ did not occur and for seven relative formulas the ‘number n drop-out ranking paradox’ did not occur. Second, for twelve formulas the ‘number 1 drop-out ranking paradox’ is more likely and for seven formulas the ‘number n drop-out ranking paradox’ is more likely. Third, for formula 16, which is a relative formula, neither ‘number 1 drop-out ranking paradox’ nor ‘number n drop-out ranking paradox’ occurred. Finally, for fourteen relative formulas the likelihood of both paradoxes occurring is below 1%.

Shape of an indifference curve

An indifference curve represents all combinations of price and quality that will receive the same score according to a formula (Chen, 2008). We analyzed the shape of the indifference curves (at different weights for price and quality) for 23 formulas. It is impossible to plot indifference curves for the other five formulas because they require some extra input such as reference price or price range, which is not available in our dataset. With price indicated on the horizontal axis and quality on the vertical axis, the indifference curves can be straight, concave or convex. The marginal rate of substitution of quality for price is defined as the price an economic agent is willing to pay to obtain one additional unit of quality. If the indifference curve is a straight line, then the marginal rate of substitution of quality for price is constant and so every unit of quality is worth the same amount of money. If the curve is concave, then the marginal rate of substitution is increasing which means that consecutive units of quality are valued more and more. If the curve is convex, then the marginal rate of substitution is decreasing which means that consecutive units of quality are less and less valued. In the fifth column of Appendix A, we list using three different sets of weights of price and quality (50-50; 60-40, and 40-60) whether the indifference curves for each formula are straight, concave or convex. This was done to compare formulas under the same conditions and does not provide a general proof of the shape of the indifference curve.

Protection against extremely high price

In certain situations, the ranking of a bid may become independent of its price. For example, when the weight of quality is very high and one bidder knows that he can offer a level of quality that gives him a sufficient advantage over other bidders, the bidder will win the tender regardless of how high his price is. For instance, let the maximum quality score be 90 points and the maximum price score be 10 points out of 100 points. If one bidder knows he can score more than 80 points on quality and that the quality score of the other bidders will not exceed 70 points, then he can charge anything he wants and still be ranked number 1. In other words, a formula that does not provide protection against an extremely high price is one for which under certain circumstances, ranking of a bid ranked number 1 does not depend on its price.

To investigate this issue, we performed an empirical study using our dataset of 357 tenders. We applied 21 formulas to each tender with the quality weight set to 80% (which is not unrealistic since our dataset contained tenders with this quality weight) and generated 357 initial rankings. Then, we increased the price of the top-ranked bid 50-fold; 100-fold; 200-fold; 400-fold; and 800-fold. We recalculated the scores and generated 357 rankings for each price increase. We compared the initial rankings with the rankings after the price had increased. If the bid ranked as number 1 in the ranking with the price increase is the same as the bid ranked as number 1 in the initial ranking, then we assume that the formula does not provide protection against this extreme price increase. We observe that a 50-fold price increase gives almost the same results as an 800-fold price increase. In the sixth column of Appendix A, we report the percentage of cases where a given formula ranked the bid with the 50-fold price increase (which can already be considered an extremely high price in a normal tender) as the best bid. It seems that when the quality weight is set to 80%, 15 of 21 tested formulas do not provide full protection against an extremely high price.

How does a formula reflect weights of price and quality?

In this section, we analyze data from four real tenders to show how formulas differ in how they emphasize price versus quality. We selected four tenders from our dataset, each with two bidders. We selected a tender with one bid with high quality and high price, and one bid with low quality and low price. We selected three more tenders, each with two bids with similar prices: One with two low quality bids, one with two high quality bids, and one with significant differences in quality. Table 1 shows prices and quality levels (as a percentage) for all four tenders.

Table 1 – The four real-world tenders selected for analysis

	Bid	Quality	Price
Tender 1 - HiQ vs. LoQ, different P	A	0.4000	352250
	B	0.7000	1301500
Tender 2 - Both LoQ, similar P	C	0.3784	140086.34
	D	0.3818	142065.42
Tender 3 - Both HiQ, similar P	E	0.9438	533613
	F	0.9688	567860
Tender 4 - HiQ vs. LoQ, similar P	G	0.2863	1.97
	H	0.7538	2.05

For 20 formulas, we calculated the “tipping point”, defined as the percentage weight of price, above which the lower price-lower quality bid becomes the best bid. Note that these are bids A, C, E, and G in Table 1. For low weights of price, starting at 0% price and 100% quality, the higher price-higher quality bid will win. At some higher weight for price, the lower priced bid will become best bid. For example, a tipping point of 29.13% means that the higher-quality bid is ranked best bid if the weight of price is between 0% and 29%, and the lower-price bid is ranked best bid if the weight of price is 30% or higher.

From columns 7 to 10 of Appendix A, it is clear how for each formula different weights of price and quality define which bid is ranked as best bid. For example, in column 7, some formulas rank the lower-price bid as number 1 only when the weight of price is higher than 37%, while some others already do so with a price weight above 10%. In the ‘low quality, low quality, same price level’ scenario (column 8) some formulas rank the lower priced bid as number 1 for weights of price above 1%

(i.e. for nearly the entire range of possible price weights) while some others only from a price weight above 55%.

There are four formulas that seem much more sensitive to price than the other formulas. This effect is most pronounced in the ‘high quality, low quality, same price level’ scenario (column 10). In this scenario, one would expect that only a very high weight of price (>90%) would make the bidder with the much lower quality and slightly higher price win the tender. Not so for four of the 20 formulas, since their tipping point is around 30%! When analysing these formulas, one can see why. These formulas incorporate not only the best (lowest) price, but also the worst (highest) price and therefore depend on a bid spread. The difference between the best and worst price defines the price evaluation range. If these best and worst price do not differ much, the formula becomes very sensitive to price. Only in the ‘high quality, low quality, different price level’ scenario the tipping point is comparable with the other 16 formulas. This behaviour can also be explained: In this scenario the highest and lowest price differ a lot, so the price evaluation range becomes very large making the sensitivity to price much lower.

Conclusion

A balanced and properly functioning bid evaluation formula to choose the economically most advantageous bid is a critical task for any buyer. Listing and measuring against award criteria is an intense process getting abundant attention from both procuring entities and bidders, often debated and even contested in court. The formula itself often gets far less attention; formulas are often chosen without carefully analyzing their properties. Our experience is that most often tendering entities are unaware of what alternatives exist to a formula they use.

This research is the first in its kind listing 28 different formulas used in procurement practice and analyzing them on specific dimensions such as the choice between relative and absolute formula, likelihood of a ranking paradox, the shape of indifference curves associated with the formula, likelihood of not providing protection against extremely high price and how a formula reflects weights of price and quality.

When choosing between an absolute and a relative formula, tendering entities should consider the risk of a mismatch between reference price or price range and market prices and the risk of a ranking paradox. All absolute formulas we study, except three, need some extra input such as reference price or price range. This creates the burden for the tendering entity of requiring pre-tender market price knowledge and with it, the risk of deviations between expectation and reality.

As for the ‘ranking paradox’ (Chen, 2008), we consider ‘number 1 drop-out ranking paradox’ and ‘number n drop-out ranking paradox’. ‘Number 1 drop-out ranking paradox’ seems to be more intuitive. If a bid ranked as number 1 pulls out, it may not always be the case that a bid ranked as number 2 becomes number 1. Hence its occurrence does not seem significant. On the other hand, ‘number n drop-out ranking paradox’ is counterintuitive. It creates the possibility of collusion among bidders since irrelevant bids can influence which bid will be ranked number 1. In our study of 280 real tenders, we show that the practical occurrence of the ranking paradox for most formulas is very small. Formula 3 has a relatively higher likelihood of a ‘number n drop-out ranking paradox’ occurring. For the other formulas, ranking paradox seems to be more of a theoretical notion than a real risk in practice.

We believe that special attention should be given to formulas with either concave or convex indifference curves. A linear relationship between price and quality implies that incremental units of quality have a constant value. A non-linear relationship implies that units of quality vary in value depending on level of quality of the individual offer. Concave graphs imply that the buyer values consecutive units of quality more and more, while Convex graphs imply the opposite. We believe that a quality score derived from a predefined weighted multi-criteria analysis must imply the use of a linear relationship between price and quality, or in other words, a straight indifference curve. Non-linear relationships imply that the exact same score on a specific criterion may lead to a different valuation by the buyer. This is in conflict the requirement of non-discrimination: Buyers need to value suppliers exactly the same on specific criteria if their performance is exactly the same.

Considerations about protection against extremely high price require determining what an extremely high price is. We feel that this quantity is either tendering entity or tender specific. In our study of 357 real tenders, we counted the number of cases in which formulas did not provide protection against extremely high price when the price of a bid ranked as number 1 in each tender was increased up to 800-fold. Out of 21 formulas that were evaluated on this dimension, 15 formulas proved not to provide full protection against an extremely high price.

Finally, we considered four different scenarios to demonstrate how different the 28 formulas are when it comes to reconciling the weights of price and quality. Among other factors, the outcome of the tender may not only depend on a choice of a formula, but also on the choice of weights for price and quality. We found that the choice of a formula and the choice of weights for quality and price interact to determine the outcome of the tender. Some formulas could not be tested because they require information that is particular to the tender. This paper provides suggestions for what kind of simulations the tendering entity can perform to study the behavior of these formulas. This research should help tendering entities to challenge the formula they use and / or to discover and choose a formula that best serves the goals of their organizations.

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Appendix A

	R / A *	Nbr 1 rank rev	Nbr n rank rev	Indff curve **	No prot against high price	HiQ vs LoQ, different P ***	Both LoQ, similar P ***	Both HiQ, similar P ***	HiQ vs LoQ, similar P ***
1	R	0.00%	0.08%	Cav	4.76%	29.13%	19.04%	29.19%	92.28%
2	R	1.11%	1.00%	N/A	2.66%	23.07%	0.33%	2.43%	31.85%
3	R	8.86%	7.70%	N/A	67.16%	23.07%	0.33%	2.43%	31.85%
4	R	1.11%	1.00%	N/A	2.96%	23.07%	0.33%	2.43%	31.85%
5	R	0.00%	0.58%	Str	0.56%	20.71%	18.93%	28.56%	92.14%
6	R	0.00%	0.08%	Str	2.80%	29.13%	19.04%	29.19%	92.28%
7	R	2.14%	0.00%	Str	2.52%	29.14%	19.62%	29.31%	92.29%
8	R	0.71%	0.00%	Cav	8.17%	37.00%	38.56%	29.85%	94.07%
9	R	0.71%	0.00%	Str	0.00%	24.13%	55.14%	44.40%	96.81%
10	R	1.07%	0.00%	Cav	0.00%	13.72%	38.66%	28.68%	93.85%
11	R	1.07%	0.00%	Str	5.04%	23.07%	18.83%	27.92%	91.99%
12	R	0.00%	0.58%	Str	0.56%	20.71%	18.93%	28.56%	92.14%
13	R	10.00%	1.24%	Str	0.00%	10.01%	18.83%	27.92%	91.99%
14	A	0	0	Vex	N/A	N/A	N/A	N/A	N/A
15	R	0.36%	0.08%	Str	0.00%	18.20%	31.54%	43.62%	95.82%
16	R	0.00%	0.00%	Cav	0.00%	24.13%	24.41%	35.64%	94.19%
17	R	0.71%	0.00%	Cav	5.04%	23.07%	13.96%	21.70%	89.04%
18	A	0	0	Str	0.30%	29.98%	38.94%	29.59%	96.05%
19	A	0	0	Str	N/A	N/A	N/A	N/A	N/A
20	R	0.00%	0.67%	N/A	0.28%	13.04%	0.16%	1.23%	18.94%
21	R	0.00%	0.58%	Str	0.56%	20.71%	18.93%	28.56%	92.14%
22	R	0.00%	0.08%	Str	0.00%	10.01%	18.83%	27.92%	91.99%
23	R	1.11%	0.42%	N/A	7.10%	N/A	N/A	N/A	N/A
24	A	0	0	Vex	N/A	N/A	N/A	N/A	N/A
25	A	0	0	Str	N/A	N/A	N/A	N/A	N/A
26	A	0	0	Str	N/A	N/A	N/A	N/A	N/A
27	A	0	0	All	N/A	N/A	N/A	N/A	N/A
28	A	0	0	Str	N/A	N/A	N/A	N/A	N/A

*R=relative; A=absolute

** Str = Straight; Cav = Concave; Vex = Convex; All = Straight, Concave or Convex depending on user's setting of n.

*** The percentage score indicates the "tipping point", i.e. the percentage weight for price, above which the lower priced-lower quality bid becomes best bid.

Appendix B

$Weight_{Quality}$ - weight of quality	P_{Avg} - average price of all bids
$Weight_{Price}$ - weight of price	P_{Best} - lowest price of all bids
Q_i - quality of each individual bid	P_{Max} - highest price of all bids
Q_{Best} - highest quality of all bids	P_{Ref} - reference price
Q_{Ref} - reference quality	$P_{Set\ Min}$ - lower end of predefined price range
VQ value of quality	$P_{Set\ Max}$ - upper end of predefined price range
P_i - price of each individual bid	N number of bids

Formulas

1. Lowest Bid Scoring	$Score_i = \frac{P_{Best}}{P_i} W_{Price} + Q_i W_{Quality}$
2. Highest Bid - Lowest Bid Scoring	$Score_i = \frac{P_{Max} - P_i}{P_{Max} - P_{Best}} W_{Price} + Q_i W_{Quality}$
3. Average Scoring	$Score_i = \begin{cases} W_{Price} + Q_i W_{Quality} & if\ P_i < P_{Avg} \\ \frac{P_{Max} - P_i}{P_{Max} - P_{Avg}} W_{Price} + Q_i W_{Quality} & otherwise \end{cases}$
4. Based on Bid Spread	$Score_i = \frac{P_i - P_{Max}}{P_{Max} - P_{Best}} W_{Price} - Q_i W_{Quality}$
5. Based on Average Bid	$Score_i = \frac{P_i}{P_{Avg}} W_{Price} - Q_i W_{Quality}$
6. Maximum Price Deviation Model	$Score_i = \left(1 - \frac{P_i}{P_{Max}}\right) W_{Price} + Q_i W_{Quality}$
7. Negometrix	$U_i = \frac{\left(1 - (Q_{Best} - Q_i)\frac{W_{Quality}}{W_{Price}}\right) P_{Best}}{P_i}$ $Score_i = P_i \left(\frac{\max(U_1, \cdots, U_N) - U_i}{\max(U_1, \cdots, U_N)}\right)$
8. Coventry City Council	$Score_i = \frac{P_{Best}}{P_i} W_{Price} + \frac{Q_i}{Q_{Best}} W_{Quality}$
9. European Organization for Nuclear Research (CERN)	$Score_i = 0.5\left(1 - \frac{P_i}{P_{Best}}\right) W_{Price} + \frac{Q_i}{Q_{Best}} W_{Quality}$
10. Tennet	$Score_i = P_i + P_i \left(1 - \frac{Q_i}{Q_{Best}}\right) \frac{W_{Quality}}{W_{Price}}$
11. Mercer	$Score_i = \begin{cases} \left(1 - \frac{P_i - P_{Best}}{P_{Best}}\right) W_{Price} + Q_i W_{Quality} & if\ \frac{P_i - P_{Best}}{P_{Best}} \le 1 \\ Q_i W_{Quality} & otherwise \end{cases}$
12. Scottish Government	$Score_i = \left(0.5 - \frac{P_i - P_{Avg}}{P_{Avg}}\right) W_{Price} + Q_i W_{Quality}$
13. Waterschap Brabantse Delta	$Score_i = \left(1 - \frac{P_i - P_{Best}}{P_{Best}}\right) W_{Price} + Q_i W_{Quality}$ $Score_i = \left(1 - \frac{P_i - P_{2nd\ Best}}{P_{2nd\ Best}}\right) W_{Price} + Q_i W_{Quality}$ If the price difference between the lowest bid and the 2nd lowest bid is greater than 20%, then the 2nd lowest bid gets 80% of price points of the lowest bid and the score of consecutive bids is calculated according to the right-hand formula.

14. Chen 1 $Score_i = \frac{P_i}{P_{Set\ Max}} W_{Price} + \frac{Q_{Set\ Min}}{Q_i} W_{Quality}$
15. Chen 2 $Score_i = \left(1 - 0.5 \frac{P_i}{P_{Best}}\right) W_{Price} + Q_i W_{Quality}$
16. Chen 3 $Score_i = \left(1 - 0.5 \frac{\log\left(\frac{P_i}{P_{Best}}\right)}{\log(2)}\right) W_{Price} + Q_i W_{Quality}$
17. Chen 4 $Score_i = \begin{cases} \left(1 - \frac{\log\left(\frac{P_i}{P_{Best}}\right)}{\log(s)}\right) W_{Price} + Q_i W_{Quality} & if\ \left(1 - \frac{\log\left(\frac{P_i}{P_{Best}}\right)}{\log(s)}\right) W_{Price} > 0 \\ Q_i W_{Quality} & otherwise \end{cases}$ where s is a user-defined parameter.
18. Argitek $Score_i = \frac{P_i}{Q_i^{\frac{W_{Quality}}{W_{Price}}}}$
19. Telgen $Score_i = \frac{P_{Set\ Max} - P_i}{P_{Set\ Max} - P_{Set\ Min}} W_{Price} + Q_i W_{Quality}$
20. Pauw & Wolvaardt $Score_i = \frac{P_{Max} - P_i}{P_{Max} - P_{Avg}} W_{Price} + Q_i W_{Quality}$
21. Based on the Average Price $Score_i = \left(1 - \frac{P_i - P_{Best}}{P_{Avg}}\right) W_{Price} + Q_i W_{Quality}$
22. Based on the Lowest Price $Score_i = \frac{2P_{Best} - P_i}{P_{Best}} W_{Price} + Q_i W_{Quality}$
23 Score by Rank $Score_i = p\, W_{Price} + Q_i W_{Quality}$ p is the score on price. The highest price bid earns 0 and the lowest priced bid 1 point on the price score. All other price scores are placed at equal increments between 0 and 1 according to their ranking on price.
24. Kuiper 1 $Score_i = P_i - \frac{W_{Quality}}{W_{Price}} P_{Ref} \left(1 - \frac{Q_{Ref}}{Q_i}\right)$
25. Kuiper 2 $Score_i = \frac{P_i}{Q_i}$
26. Kuiper 3 $Score_i = \left(2 - \frac{P_i}{P_{Ref}}\right) W_{Price} + \left(\frac{Q_i}{Q_{Ref}}\right) W_{Quality}$
27. Kuiper's Superformula $Score_i = \sqrt[n]{W_{Price}\left(\frac{P_i}{P_{Q=1}}\right)^n + W_{Quality}\left(\frac{1 - Q_i}{1 - Q_{P=0}}\right)^n}$ where n is a user-defined parameter
28. CROW Value Based $\mathrm{Score_i} = \mathrm{P_i} - (\mathrm{VQ} \times \mathrm{Q_i})$